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Two-dimensional effects in double-diffusive convection

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The limitations of existing one-dimensional experiments on double-diffusive convection are discussed, and a variety of new two-dimensional phenomena are described. We have used the sugar–salt system and shadowgraph photography to make exploratory studies of motions which can arise in a fluid with two smooth, opposing, vertical concentration gradients, with and without horizontal gradients. Many different effects have been observed, the most important of which are the following. (*a*) In the 'finger' case, local disturbances can propagate rapidly as wave motions, which cause a simultaneous breakdown to convection over large horizontal distances. (*b*) Layers formed in the 'diffusive' sense overturn locally to produce fingers, but propagate more slowly, as convective rather than wave motions. (*c*) A series of layers, separated by diffusive interfaces, can become unstable, in the sense that successive layers merge in time as their densities become equal. (*d*) The presence of horizontally separated sources of water of similar density but different T, S characteristics can lead to the development of strong vertical gradients and extensive quasi-horizontal layering.

Most of our results are qualitative, but it is hoped that they will stimulate further quantitive work on each of the new processes described. It is already clear that much more needs to be done before the mechanism of formation of layers observed in the ocean can be regarded as properly understood.

1. Introduction

There has been a great deal of interest recently in observations of microstructure in the deep ocean. These have revealed extensive layers of relatively well-mixed water, separated by interfaces across which the gradients of temperature and salinity are much larger. In some cases there is a systematic association between the temperature and salinity steps, with both properties increasing or both decreasing with depth. So many observations of this kind have now been reported that it is unnecessary to describe them again in detail, but we can refer to the striking examples due to Tait & Howe (1968, 1971) and Neal, Neshyba & Denner (1969) in which the stratification is in the two possible senses.

It is attractive to interpret such measurements in terms of double-diffusive convection processes, which can draw on the potential energy of the component which is heavy at the top (Turner & Stommel 1964; Turner 1965, 1967). Laboratory experiments have revealed qualitative features similar to those found in the ocean, and the technique of using two solutes with different diffusivities instead of heat and salt has proved very useful (Stern & Turner 1969). Most of these experiments, however, have been one-dimensional in form. That is, they have been concerned with horizontally uniform vertical transports of properties across established interfaces, or have studied the successive formation of layers, for example due to uniform heating of a salinity gradient from below (Turner 1968). It is the purpose of the present paper to report the results of a series of exploratory, mostly qualitative, laboratory experiments in which various two-dimensional effects are included for the first time.

As was pointed out recently by Huppert & Turner (1972), it is difficult to find geophysical situations where we can be sure that the physical processes are sufficiently similar to those studied in the laboratory for the same quantitative relations to be applicable. One such case is Lake Vanda in the Antarctic (Hoare 1966), a salt-stratified lake whose temperature structure can be explained using Turner's (1965) one-dimensional laboratory results. In the ocean, however, horizontal advection is nearly always an important factor, and its neglect in previous laboratory experiments makes it hard to justify at this time the kind of detailed comparison suggested by Fedorov (1970).

Strong layer formation in fact often seems to be associated with the intrusion of one water mass into another (for instance layers are prominent above and below the Mediterranean outflow into the Atlantic). This suggests several types of problem in need of further study. In general we should aim to learn more about the stability and modes of breakdown of a fluid containing two smooth, opposing, vertical gradients (which could be set up by an intrusion), rather than concentrating as in the past on the case of a flux of one property into a stable gradient of the other. Second, the possibility that local disturbances of various kinds could propagate horizontally must be examined. Finally, we should study the effect of horizontal gradients of properties, produced by water masses of different origin, on the stability of a double-diffusive system. Each of these will receive some attention in this paper though the variety of different phenomena which can occur is remarkable, and each deserves a more extensive quantitative investigation. We do not claim that the range of conditions explored here has exhausted all the relevant possibilities, and indeed it seems likely that other important effects remain to be discovered.

2. Previous related experiments

Several experiments involving horizontal non-uniformities of various kinds have already been reported in the literature. Some of these have recently been summarized by Turner (1973), but they will be mentioned again here so that they will be more readily available for comparison with the new results to be described.

Thorpe, Hutt & Soulsby (1969) and Chen, Briggs & Wirtz (1971) have studied the behaviour of a fluid stratified with a solute when it is heated and cooled through vertical side walls. Thorpe *et al.* concentrated on the case of a narrow gap, and made a linear stability analysis which predicted the relation between a critical thermal Rayleigh number Ra_c based on the lateral temperature gradient, and a solutal Rayleigh number R_S based on the initial vertical solute gradient. Their experiments confirmed that such a critical value Ra_c exists and that the resulting motion consists of a series of inclined cells, whose height they calculated, extending right across the gap. Somewhat better agreement with these same experiments was obtained by Hart (1971), whose theory took into account the mean flow due to the side-wall heating and included more realistic boundary conditions.

Chen *et al.* were concerned, on the other hand, with the initiation of instability along a single heated wall in a tank with such a wide gap that the presence of the opposite wall was not felt by the convecting fluid till much later. They showed that the important parameter is then the thermal Rayleigh number based on a length scale

$$l \approx \frac{\alpha \Delta T}{\beta dS/dz} \tag{1}$$

(where ΔT is the imposed temperature difference and α and β are the 'coefficients of expansion' for temperature and salinity changes), which is the height of rise of a heated parcel in the initial stratification dS/dz. This critical Rayleigh number was shown experimentally to be $15\,000 \pm 2500$. At supercritical Rayleigh numbers, cells appeared simultaneously all along the heated wall, and grew laterally away from it; their height was comparable with the scale l defined by (1). The motion in the fully developed state (after the cells had reached the opposite cooler wall) was shown using streak photography to consist of an upflow near the heated wall, feeding an outflow which extended, with a slight downward tilt, right to the opposite wall. The return flow was antisymmetric with this, producing a closed circulation in the same sense in each cell, with a comparatively quiescent region in the centre and high shears across the cell boundaries. Thorpe *et al.* (1969) also made some experiments using a wide tank, and documented reversals of the vertical temperature and solute gradients which can be explained in terms of this same circulation pattern.

An experiment which suggests the kind of motion which could originate in the interior of a fluid, owing to horizontal gradients, was described by Stern & Turner (1969). They used a 'dam break' technique in a long channel, with warm salt water initially separated from cold fresh water of nearly the same density by a central vertical barrier. In addition to being a convenient way of setting up a two-layer system with small initial density differences, this configuration shows how an initial disturbance can be amplified by the motion it produces. When the dam is removed, the vertical interface may tilt in either direction; double-diffusive transfer across it (in either the 'finger' or 'diffusive' sense) will tend to increase the density difference between the layers, and so reinforce the initial direction of motion. In this experiment, of course, the vertical scale of the layers is imposed, rather than being predicted by a stability argument.

Another type of non-uniformity which has received some attention is the variation of depth of a convecting layer. Gill & Turner (1969) have shown that, in such a region, large-scale quasi-horizontal motions can be set up even when the buoyancy flux across the horizontal boundary of the region is uniform. The effect is a purely geometrical one; a given flux across the upper boundary or interface causes the density of the lower layer to increase most rapidly in its shallowest part, where there is less dilution. This sets up a horizontal pressure gradient, driving a circulation in the sense which includes a flow down the slope. In a two-layer double-diffusive system there is also a circulation in the same sense in the upper layer even when its depth is uniform. This implies that there is a strong shear across the interface, and it can be explained in terms of the change in buoyancy flux produced by the variation in concentration difference with position along the interface.

Circulations of this kind have been observed in the laboratory with sugar and salt layers set up in both the 'finger' and 'diffusive' senses (see Turner 1973). Note that in this case the flow of heavy fluid down the slope goes right to the bottom, and sets up a stable stratification there. Moreover this flow reverses the relative concentrations in the lower region, so that, for example, diffusive layers are formed in a fluid which originally contained salt fingers everywhere. This reversal of properties is a prominent feature of some of the ocean observations (e.g. Tait & Howe 1971). It appears also in several of the new laboratory observations described later, and is characteristic of convection situations in which there is a systematic overturning without thorough mixing throughout the depth of the fluid.

Finally, it is of interest to report similar effects observed in (previously unpublished) experiments in which a sloping interface produces the non-uniformity of depth, rather than a solid sloping boundary. Figure 1 (plate 1) shows a three-layer sugar–salt system, in which the concentrations were chosen so as to make the buoyancy flux into the intermediate layer from above much larger than that from below. Any tilting of the lower interface sets up a circulation which continuously feeds the heaviest fluid into the deepest part of the intermediate layer, thus increasing the initial tilt. The final result, when the buoyancy flux through the upper interface has made the densities on the two sides of the lower interface equal, is a breakthrough of the intermediate layer into the lower fluid at the deepest point. This process too is related to one of the new experiments discussed below.

3. Experiments with opposing vertical gradients

The feature common to all the new experiments is the existence of opposing vertical gradients of two properties. One such experiment was reported by Stern & Turner (1969, p. 508), but their observations raised more questions than they resolved. The observations reported in the following three sections are a natural continuation of that work, but were carried out in a longer tank, 90 × 7 cm in cross-section, to allow two-dimensional effects to be studied.

Again we have used sucrose and common salt as convenient solutes, so as to avoid unwanted side-wall heating or cooling effects. These will be denoted respectively by S, the property with the lower molecular diffusivity κ_S, and T,

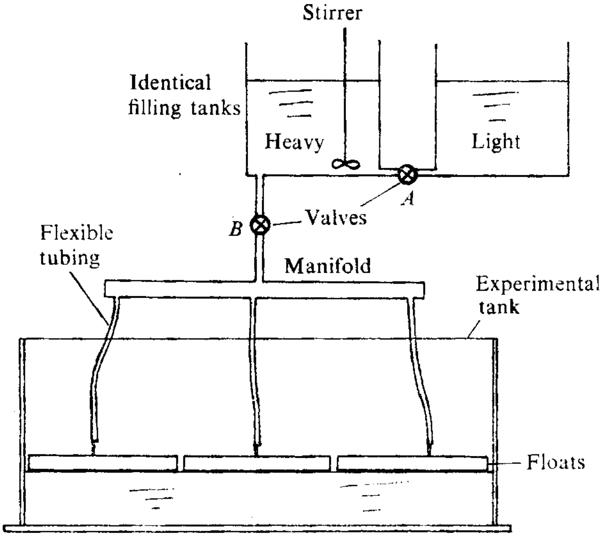

FIGURE 2. Sketch of the filling process using the 'two bucket' method and three floats.

the property with the higher diffusivity κ_T ($\kappa_S/\kappa_T \approx \frac{1}{3}$ for sucrose and NaCl). The experimental tank was filled by means of the two-bucket technique described by Oster (1965), and sketched in figure 2. Using one container filled with sugar solution and one with salt solution, we set up gradients of the two properties over a typical depth of 25 cm, with the concentration of one solute having a maximum at the bottom, decreasing to zero at the top, and the other a maximum at the top, decreasing to zero at the bottom. Our previous experience of setting up gradients of a single property suggests that the distributions were very close to linear. The heavier fluid was introduced first, and the mixed solution of decreasing density put in on top of this through one or more floats made of a foam plastic rim with a porous bottom. The specific gravity of the original solutions, measured by weighing samples, was in the range 1·13–1·14, while the difference between the two was much smaller (0·003–0·012). The filling time ranged from about 20 min to an hour.

The subsequent motion in the tank was viewed and photographed using a shadowgraph method. The light source was an uncollimated beam from a small projector lamp located about 5 m away, and the screen consisted of tracing paper attached to the front wall of the tank. Both still and time-lapse pictures were taken, and the main purpose of the present paper is to document various new phenomena revealed by the former. It should be remembered, however, that much of our understanding of these has come from viewing the time-lapse movies, speeded up from 12 to 150 times.

4. Instability of a field of fingers

When the sugar (S) was a maximum at the top and the salt (T) a maximum at the bottom of the tank, 'fingers' were observed to form rapidly throughout the tank, even during the filling process. With the large individual concentration

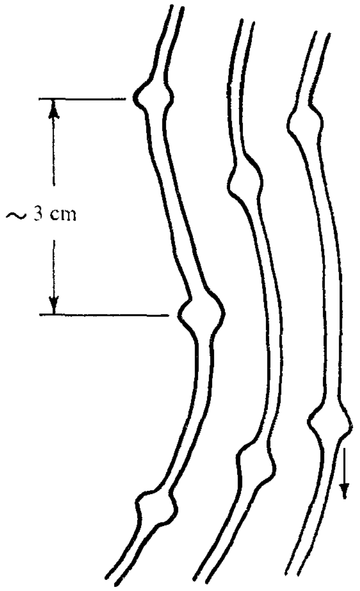

FIGURE 3. Sketch of the small-scale 'lumpy' disturbances on individual sugar–salt fingers, which develop when there are large individual concentration gradients and a small net density gradient.

gradients and the small net density gradients used in these experiments, the field of fingers always appeared less stable and regular than has previously been observed (compared with, say, figure 1 in Shirtcliffe & Turner 1970). The instability took different forms in different circumstances, which we can summarize as follows.

Prominent in the early stages of all such finger experiments were small-scale 'lumpy' disturbances on individual fingers, which are difficult to photograph, but are sketched in figure 3. These look like the vortex caps which form at the front of thin viscous plumes, except that they occur along the length of the fingers, with a typical vertical spacing of 3–4 cm, and randomly through the volume of the fluid, rather than just at the ends of the columns. They suggest that the motion is not of the equilibrium type described by theories such as that of Huppert & Manins (1973), but that the vertical motion driven by the potential energy in the S distribution is too rapid to be balanced by laminar viscosity and diffusion alone. If the tank is allowed to 'run down' for a long time (of order one hour), then a much smoother pattern of vertical fingers is achieved.

When horizontal non-uniformities, as well as vertical gradients, were deliberately introduced by filling the tank through a single float at one end, another kind of motion was observed. This consisted of unsteady alternating motions in layers, rather like that found by Chen *et al.* (1971, figure 5) in their experiments on heating a salinity gradient from the side (see figure 4). An explanation can be given in similar terms to that presented in § 2 (except that the motion is unsteady, and it is not now initiated by a flux through the end wall), and more will be said about it in § 7. These motions could readily be seen on the time-

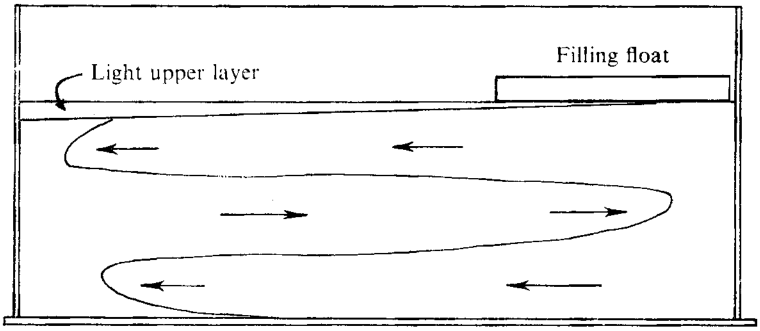

FIGURE 4. Sketch of the alternating motions in layers which develop during the filling of the experimental tank from one end with sugar above salt. These are associated with the horizontal gradients of the two properties, and should be compared with the motions sketched in figure 15. Also shown is a thin layer immediately below the free surface, which can often be traced back to the beginning of the filling process. If the first salt solution to flow in is slightly diluted by fresh water already in the tank, differential transport through the 'diffusive' interface bounding it below can keep this layer lighter than any fluid added subsequently. Such a surface layer is seen in the photograph in figure 5.

lapse films because of the systematic, horizontally correlated distortions of the fingers out of the vertical. We do not have a convincing record on the still frames; for this purpose, streak pictures of particles in the flow would be more satisfactory than the shadowgraph technique.

These distortions of the fingers suggest the 'collective instability' mechanism described by Stern (1969), but provided that the filling was completed at a constant rate, no further breakdown or overturning was observed. That is, the wave motions became stabilized at a finite amplitude, and the net density distribution remained smoothly increasing with increasing depth rather than overturning and mixing in layers as Stern predicted. After about half an hour these motions had died away completely, and undistorted fingers filled the whole tank. However, when a disturbance was introduced by altering the flow rate during the filling process (still using one float), a mixed layer formed near the input and propagated with a well-marked front down the channel (as shown in figure 5, plate 2). Eventually a convecting layer of constant depth, bounded above and below by fingers, extended across the whole tank. This agrees with the observations reported by Stern & Turner (1969), which suggested that a finite amplitude disturbance can cause a double-gradient system to break down and convect, although it would have remained stable to small internally generated disturbances.

Similar experiments were carried out using three floats for filling, covering the whole of the surface area of the tank, so that no horizontal gradients of S and T were introduced. In this case the fingers remained vertical, and extended from the top to the bottom of the tank. They exhibited the small-scale lumpy instabilities, but none of the wavelike motions in thin layers, which are thus clearly associated with the horizontal gradients of properties. Horizontal non-uniformities were introduced into this system in another way, by causing a disturbance mechanically at one end of the tank, either by filling it past a wedge mounted on the end wall, or by raising a flap after the filling was completed. An instability could now develop quite differently, as shown in the two photographs in figure 6 (plate 2). The local disturbance propagated rapidly right across the tank in the form of nearly horizontal wave motions confined to the level of the wedge, and then there was a rapid overturning to give convection almost simultaneously at all positions along the affected region. This convecting layer then grew slowly at the expense of the fingers on either side of it; groups of fingers being sheared off and feeding the convection can be clearly seen in the photographs.

Our ability to produce such a convecting layer depended strongly on the time at which the disturbance was introduced. The behaviour described above occurred only when the flap was raised shortly after (or during) the filling of the tank. When the fingers were allowed to 'run down' for as little as 5 min after filling was completed, even vigorous stirring with the flap did not cause a convecting layer to form. Figure 7 (plate 3) shows the two effects very clearly. At the bottom of the tank there is a convecting layer, produced early in the experiment by raising the flap away from the end wall into a downward sloping position. The flap was then given a further upward slant as shown, but no convecting layer could be made to form at this new level.

One approach to the finite amplitude stability problem posed by these experiments was sketched by Stern & Turner (1969), but this is worth further study with these new observations in mind. The breakdown depends not only on creating a large enough initial disturbance, but on the magnitude of the destabilizing S gradient in relation to the net density gradient. In the early stages of an experiment, energy can evidently be fed from the finger convection into the larger-scale wave motions and thence into the overturning. At later times, the fingers act to suppress the wave motions, and they are quickly damped.

5. Instability of a diffusive system

A similar series of experiments was carried out with the sugar and salt gradients in the 'diffusive' sense, i.e. with the salt concentration (T) a maximum at the top and sugar (S) a maximum at the bottom, and a small net density gradient. Again, filling through one or three floats was tried.

In this case, surprisingly, the smoothest distributions were obtained using a single float at one end of the tank. This seemed to be possible because the vertical transports of T and S were much slower than in the 'finger' experiments. The newly introduced fluid of a particular density could thus spread out horizontally over the whole of the tank without a substantial change in its properties, and the net result was a vertical gradient with little horizontal variation. When three floats were used, on the other hand, it was difficult to keep the inflow rates precisely equal, and small horizontal differences in density and composition were inevitably set up. These led to the formation of a complicated array of mutually intruding layers, which finally overturned to give an irregular stack of convecting layers, separated by sharp 'diffusive' interfaces. (See figure 8, plate 3.) This was much too complicated and unrepeatable for systematic study, and all the rest of the experiments described below were set up using a single float.

Starting with smooth gradients, and fluids at room temperature, the 'diffusive' systems we studied could remain stable for several hours when contained by vertical walls. In this case the surfaces of constant concentration of both T and S were normal to the boundaries, so that the no-flux condition was satisfied, and there was no tendency for horizontal gradients to be set up (as there is in the case of side-wall heating, discussed in § 2). In the several photographs in figure 9 (plates 4 and 5) there are in fact some thin faint layers visible which can be attributed to this cause. These did not influence the formation of layers described in this section, however, as evidenced by the fact that in one experiment we deliberately stirred the fluid gently to destroy the layers and the same phenomena were observed.

A rapid change occurred when a sloping boundary was inserted through the whole depth of the tank, by sliding a thin metal plate into slots milled in the front and back walls. The sequence of events is shown in the photographs in figure 9, and can be explained as follows. Molecular diffusion distorted the equal-concentration surfaces so that they became normal to the slope, rather than horizontal. This upset the hydrostatic equilibrium; with a single solute producing the stratification, a 'buoyancy layer' of constant thickness would develop, in which molecular diffusion can be balanced by a slow upward flow along the plate (Phillips 1970). With two components, as in our experiments, a steady flow of this kind is not possible. Just above the sloping boundary a downflow formed, and above it an upslope counterflow, which did not continue indefinitely, but turned outwards to form a series of approximately equally spaced layers which developed fairly uniformly all along the plate. These advanced with well-defined fronts away from the slope, affecting the environment only when a layer had extended into the region in question; there was no sign of any wavelike propagation in this case.

It was clear that the length of the sloping plate had an important influence on this process of layer formation. The flow near a short inclined flap was like that described above, but its global effect was small. In contrast with the 'finger' case described in § 4, layers did not propagate unless the flap was several layer thicknesses long. At the left of figure 8 (*a*), for example, there is an inclined flap whose length is of the order of the layer thickness; this has not produced any systematic layering which can be distinguished from the irregular structure due to the non-uniform filling.

We observed that the mechanical disturbances caused by sliding in the sloping boundary had died away well before the systematic upflows and downflows developed, so that our purely diffusive explanation of the effect of the slope seemed a reasonable one. To dispel any doubts, we established opposing gradients in a tank having a fixed sloping boundary, introducing the new fluid on top as before. The gradients were slightly nonlinear, since the cross-section increased from bottom to top, but the length of the tank (85 cm) was much larger than the depth of water (20 cm) so the distortion was small. Exactly the same sequence of events was observed as in the case where the boundary was inserted after the tank was filled, except that layers formed sooner at the bottom, where the fluid had been in contact longest with the sloping boundary (see figure 10, plate 6).

Another interesting feature observed behind each advancing front was a local reversal of the relative vertical gradients to produce 'fingers', even though the original smooth distributions were in the opposite 'diffusive' sense. This was associated with a characteristic circulation in the layers, out along the top and back towards the sloping boundary at the bottom. Each layer sloped downwards as it left the wall, and then moved slightly upwards as it extended further. (The sense of all these motions was reversed in the layers originating under the sloping plate.) At a much later time, a series of sharp diffusive interfaces remained, with a circulation in the opposite sense (corresponding to flow down the sloping boundary) within each of the layers above the plate. That is, the slope then had a purely 'geometrical' effect on the flow, such as that described in § 2 for the two-layer system, rather than the local effect on the diffusive boundary condition which initiated the instability. There was a mean flow in the whole tank which caused the layers to extend further from the slope at the bottom and inhibited their extension near the surface.

When the fronts had extended a long way from the sloping boundary where they had begun, it became clear that the motion near them could no longer be regarded as driven from behind, i.e. by the outflow from the slope. Often a new burst of activity grew spontaneously, and the front began to advance more rapidly into its previously undisturbed surroundings, as shown on the right of figure 11(*a*) (plate 7). Sometimes, too, a series of new layers and fronts formed above this, each of them exhibiting the finger structure which indicated a systematic overturning. The depth of all these layers was about the same as that of those formed originally (about 2–3 cm), but we have as yet no definite information as to what determines this scale in a disturbed double-gradient system.

6. Development of a series of diffusive layers

In all the experiments described so far, we took care to have the working fluids at room temperature, to avoid unwanted side-wall heating effects. For the runs reported in this section, we deliberately introduced a temperature difference of a few degrees, to provide a controlled disturbance to an otherwise stable double-gradient 'diffusive' system.

The behaviour is illustrated by the sequence of pictures reproduced in figure 12 (plates 8 and 9). About half an hour after filling of the tank was completed, a series of sharper gradients became visible on the shadowgraph. These were more or less coherent along the length of the tank (but with occasional blank areas) and had a typical vertical spacing of about 1 cm. At first it was thought that they arose directly as a result of the heating, as in the experiments referred to in § 2, but the length scale was not sensitive to the temperature difference over the range 1–3 °C. It now seems more likely that the temperature differences just provided a finite disturbance sufficient to release an instability which was driven essentially by the solute distributions.

It was certainly true at later times that the sugar–salt convection was responsible for sharpening the interfaces and driving convection in each of the layers between them. As the experiment continued, the convective velocities increased while the layers became more uniformly mixed. The system of layers then began to become unstable in another way, owing to the mechanism described by Huppert (1971). If the buoyancy fluxes across the two interfaces bounding a layer are unequal, the density can gradually change, so that it approaches the density of one of the adjoining layers and the two merge. Successive stages of this process are shown in figure 12. Note that merging occurred at different rates at different horizontal positions, which makes it appear that there are 'waves' on the interfaces. This is a purely kinematic effect, however, and not associated with actual vertical displacements. After about four hours (in this experiment) the interfaces became nearly horizontal, but it was not until much later that the 'final' state was attained. Further merging took place overnight to give a layer depth of about 4 cm in the centre of the fluid, indicating that there had been two stages of merging of the original 1-cm-deep layers.

Huppert (1971) made a definite prediction of the conditions under which this kind of instability of a multi-layer system can develop. He showed that the system is stable, with equal fluxes through all the interior interfaces, when the ratio of fluxes F_S/F_T is constant, but is unstable otherwise. This was related to the experimental result of Turner (1965), who showed that the flux ratio in the heat-salt case is constant when the density ratio $R_\rho = \beta\Delta S/\alpha\Delta T$ is sufficiently large, but that it becomes a function of R_ρ for $R_\rho < 2$. In the salt over sugar 'diffusive' case, Shirtcliffe (1973) has demonstrated that the flux ratio is also constant, in the range $1{\cdot}10 < R_\rho < 2{\cdot}3$. No one, however, has documented a variation of flux ratio with R_ρ for this pair of solutes, or found the value of R_ρ at which the behaviour changes. The experiments presented here provide a sensitive method for putting limits on this value, but a continuous monitoring of the two concentration gradients will be required, up to and beyond the time when no further merging is observed. All that we can say so far is that merging occurred with an initial density ratio as high as 1·06. This will thus be a lower limit to the desired value of R_ρ, while Shirtcliffe's experiments already give an upper limit of approximately 1·10.

7. Small sources with different T, S properties

Although they are included in some of the experiments already described, horizontal non-uniformities of fluid properties have played only a minor role in the above work. They have been introduced by filling the experimental tank from one end, in circumstances where the vertical fluxes of properties are large compared with the horizontal transports by advection, and there has always been an associated vertical density gradient present from the beginning of the experiment. It has been the breakdown of this vertical structure by various kinds of imposed, non-uniform mechanical disturbances which has been our main concern so far.

Now we turn to experiments in which the initial density differences are everywhere small, before one or more localized sources of fluid of very different T, S properties are introduced. It will be shown that strong vertical density gradients can be set up, combined with both vertical and horizontal T and S

gradients. Layers can then be produced and quasi-horizontal circulations sustained by mechanisms very similar to those observed when the vertical gradients were introduced directly.

Figure 13(*a*) (plate 10) shows the behaviour of a small (2 mm diameter) source of sugar solution of specific gravity 1·188, flowing out horizontally at the mid-depth into an initially uniform salt solution. This was of the same nominal density (as measured by a hydrometer), its depth was 18 cm and it was contained in a tank 30 × 7 cm in cross-section. The plume was evidently in fact slightly heavier than the fluid in the tank, and turned downwards. The flow rate was such that the motion would remain laminar in a salt–fresh water system, but with two components the result was much more violent. Because of the different diffusion rates across the boundary, the plume fluid became heavier and its immediate surroundings lighter, thus generating more rapid vertical motions, both upwards and downwards. With careful adjustment of the densities and flow rates the plume appeared to split near the orifice, with part immediately going up and part down (figure 13*b*, plate 10). In the cases illustrated the separation rate was enhanced by turbulence developing in the plumes, which increased the concentration gradients and the rate of diffusion.

The net result of this process was to generate relatively strong vertical plumes, in both senses, in which both the individual buoyancy and mass fluxes were much greater than those in the original unseparated plume. The density difference between each plume and its environment, moreover, tended to increase as a result of mixing, rather than decreasing as in a single-component flow. When each of these plumes reached the top (or bottom) of the experimental tank, it was thus much lighter (heavier) than the original fluid, and spread out there, stratifying the tank by the 'filling box' mechanism discussed by Baines & Turner (1969).

There is now the extra factor of double-diffusive convection, driven by the opposing vertical gradients of properties set up in this way, to be considered, and layers produced by the 'finger' and 'diffusive' mechanisms at the top and bottom respectively are to be seen in the photograph in figure 13(*a*). These vertical transports had another effect not envisaged in the simple filling-box model; they always tended to make the top fluid increasingly lighter and the bottom fluid heavier in the course of time (even if the input was turned off). Thus, when the input was continued, the plumes could no longer penetrate right to the top or bottom, but spread out at intermediate levels which gradually approached the level of the source. In the experiment described, the specific gravity of the fluid at the top and bottom of the tank 24 hr after the beginning of the experiment was 1·173 and 1·206 respectively. This strong stratification, with density differences very much larger than that between the original tank and input fluids, was typical of all our experiments, which were carried out also using the other possible combinations of source fluids and direction of the initial motion.

Having discovered these strong stratifying effects in a short tank, we went on to examine their influence in more extensive regions, using a tank 160 cm long and 8 cm wide. Again various initial ambient and source fluids were tried, and the behaviour near a single source at one end of the tank was very like that already described. Because of these strong local vertical transports, horizontal density anomalies were set up, which led to a spreading of the layers away from the source region right to the end of the experimental tank. The vertical transports continued as the fluid spread out, and eventually a state was reached in which there were large vertical density gradients everywhere, combined with large horizontal gradients of T and S. The latter were, however, nearly compensating, so that the net horizontal density gradient was small.

A particularly revealing experiment was carried out using two sources, one at either end of the tank. We started with a 10-cm-deep layer of fluid, consisting of a uniform mixture of salt and sugar solutions in equal parts, each with specific gravity close to 1·096. At one end was a slightly lighter source of sugar solution, and at the other a slightly heavier source of salt, adjusted to the same flow rate. In the centre, at the mid-depth, the mixture was removed through a constant-head overflow device, so that throughout the experiment the total volume, and also the total mass of T and S in the system, remained constant. As a result of this removal of fluid the density gradients at the mid-depth became very sharp, with smaller stable gradients above and below the centre.

The total vertical density difference increased rapidly at first, and then more slowly; half the final difference was achieved in 36 hr but the final 'steady state' values, specific gravity 1·117 at the bottom and 1·074 at the top, only after about a week. The structure and motion, however, remained substantially the same over the whole of this period. Their form is shown in the photograph in figure 14 (plate 11), taken at the left-hand end of the tank near the source of salt, which is by this time feeding in along the interface. The fluid already in the tank contains some sugar, so fingers appear above and diffusive layers below the source. These layers, whose number could be varied by changing the inflow rate, sloped gently upwards to the right, and were continuous to the centre of the tank. Then the structure changed to an antisymmetric form, with fingers below diffusive layers (which again sloped upward to the right).

The motion in each of these layers was also observed in the steady state. It is similar to the circulations referred to in §§ 2 and 4, and its form sheds some light on the mechanism whereby the vertical buoyancy flux, combined with horizontal T and S gradients, can sustain the quasi-horizontal motions. At the top of each layer, i.e. underneath each sharp interface, the motion was always slightly downwards, or to the left using our configuration of sources (see figure 15). The density was increasing slowly in the direction of motion because of the flux across the interface; this is consistent with a nearly uniform density in horizontal planes. At the bottom of each layer (above the interfaces) the upward flux caused a decrease in density in the direction of motion, and the motion was slightly upwards. There are thus strong shears across each sharp interface and weaker ones in the interior of the 'layers', where there must also be a small stable density gradient.

This extensive, nearly horizontal layering, with small net horizontal density differences and compensating gradients of T and S, recalls the observations by Stommel & Fedorov (1967) of similar structures in the ocean. It is tempting to

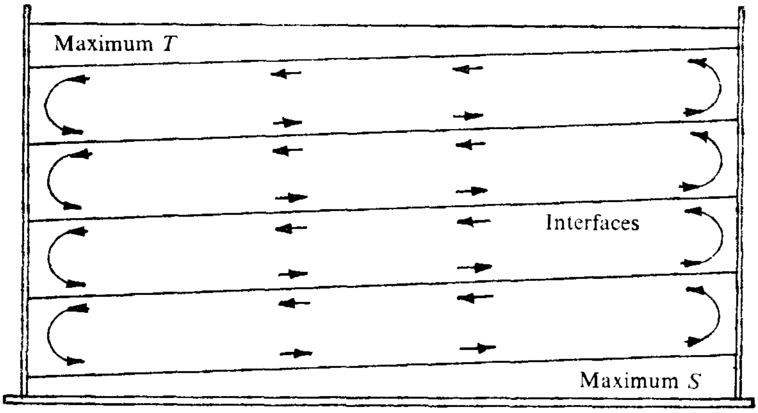

FIGURE 15. Sketch of the circulation in tilted layers, produced by a vertical buoyancy flux combined with horizontal T and S gradients.

suggest that these too could have been established by the interaction of several horizontally separated sources of water with different T and S properties, but this proposition must be followed up in more detail elsewhere.

8. Summary and conclusions

It is difficult and possibly premature to summarize adequately the observations recorded in this paper. The variety and variability of the effects seen took us by surprise, so that instead of immediately following up one or two effects in quantitative detail we were led to make the present broad survey. Obviously much more needs to be learnt about each of the processes identified here before they can be associated with particular oceanographic data, but we can already sketch some of the ways in which the results may be relevant for the ocean.

The experiments presented in § 4 show that one must keep in mind the possibility that layers will propagate rapidly in the horizontal, with a *wave* rather than a fluid velocity. This was observed in the finger situation, which may thus be quite different from the 'diffusive' case. In the latter, layers were shown in § 5 to spread more slowly by advection, extending only as the front of a layer reached a previously undisturbed region. Such layers were initiated by inserting solid sloping boundaries in our experiments, but it was also clear that they could be set up by local disturbances well away from boundaries, and could continue to propagate independently of such boundaries.

The experiments of § 6 suggest that we should be careful about identifying oceanographic measurements, of layer depths for example, with theories which relate solely to the *initial* formation of a series of layers. Whether the layers were formed originally by a one- or two-dimensional process, several stages of merging could have occurred subsequently, so making a direct comparison meaningless. It would be worth examining records of oceanic microstructure to see if any evidence of merging layers could be detected. It has already been suggested by Neshyba, Neal & Denner (1971) that the persistence and properties of layers in certain areas can be related to Huppert's (1971) criterion for stability.

In § 7 (and incidentally in previous sections), we have shown how the existence of horizontally separated sources of water with different T, S properties but the same density can lead to the formation of strong *vertical* density and concentration gradients, combined with nearly compensating horizontal T and S gradients. Associated with these is a horizontally extensive layer formation, similar to that previously attributed to one-dimensional processes, but containing circulations more nearly comparable with those observed in layers formed by side-wall heating of a stable salinity gradient. Horizontal non-uniformities of water properties certainly occur in the ocean over a wide range of scales, and it seems essential to take these carefully into account when interpreting observations of double-diffusive layers.

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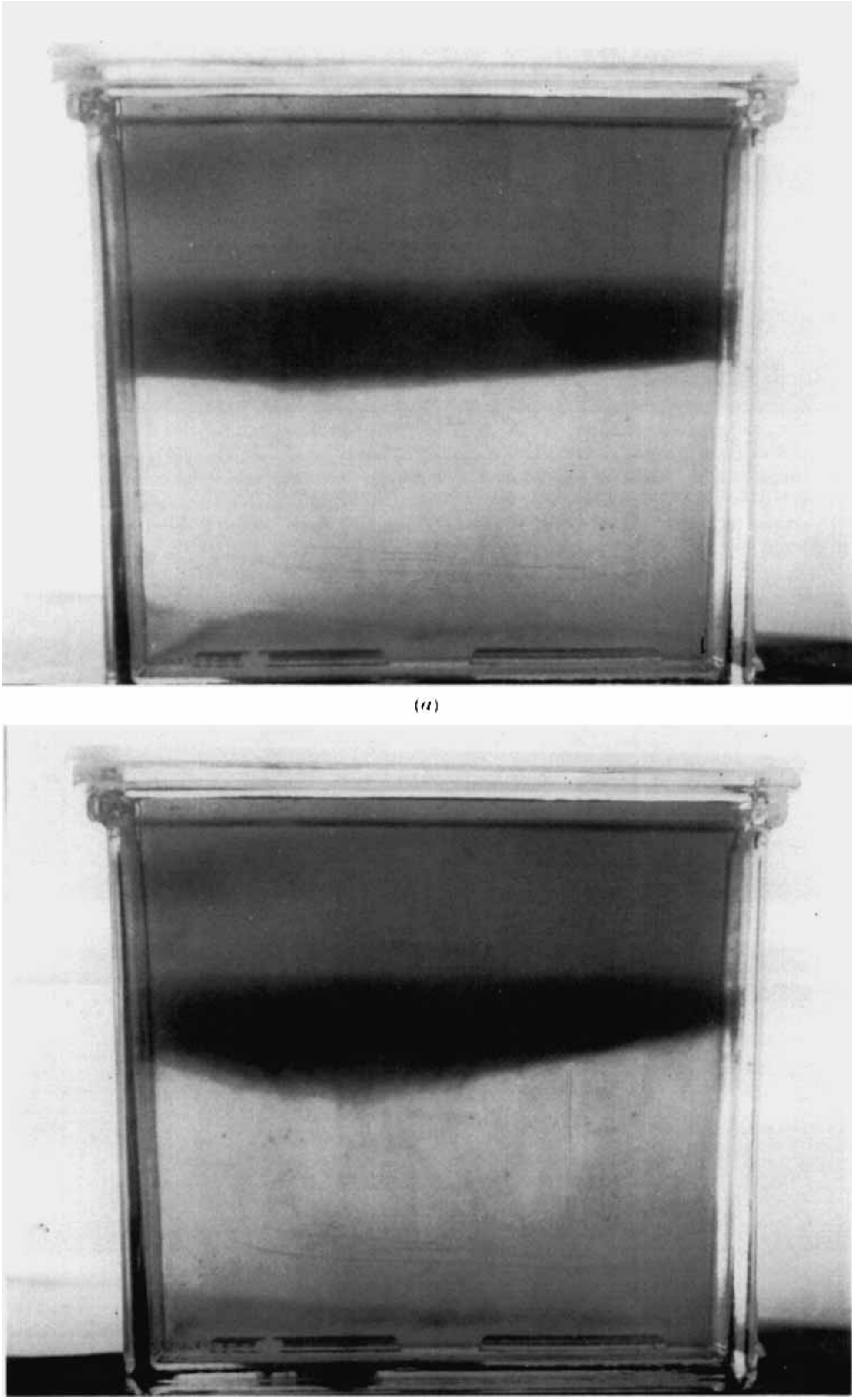


Figure 1. A three-layer double-diffusive experiment, in which the buoyancy flux through the upper interface was much larger than that through the lower. (*a*) Shows a small initial tilt on the lower interface. (*b*) This tilt set up a circulation which fed heavy fluid into the deepest part of the middle layer, thus increasing the tilt to the stage shown here. Later the intermediate layer broke through into the lower at the deepest point.

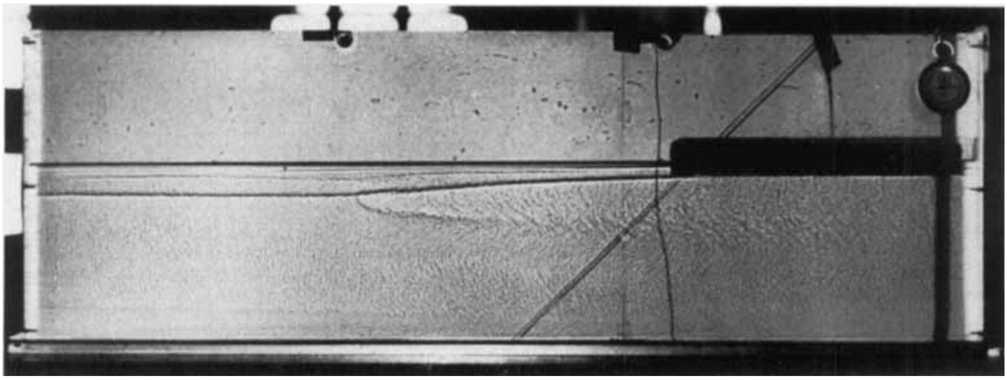
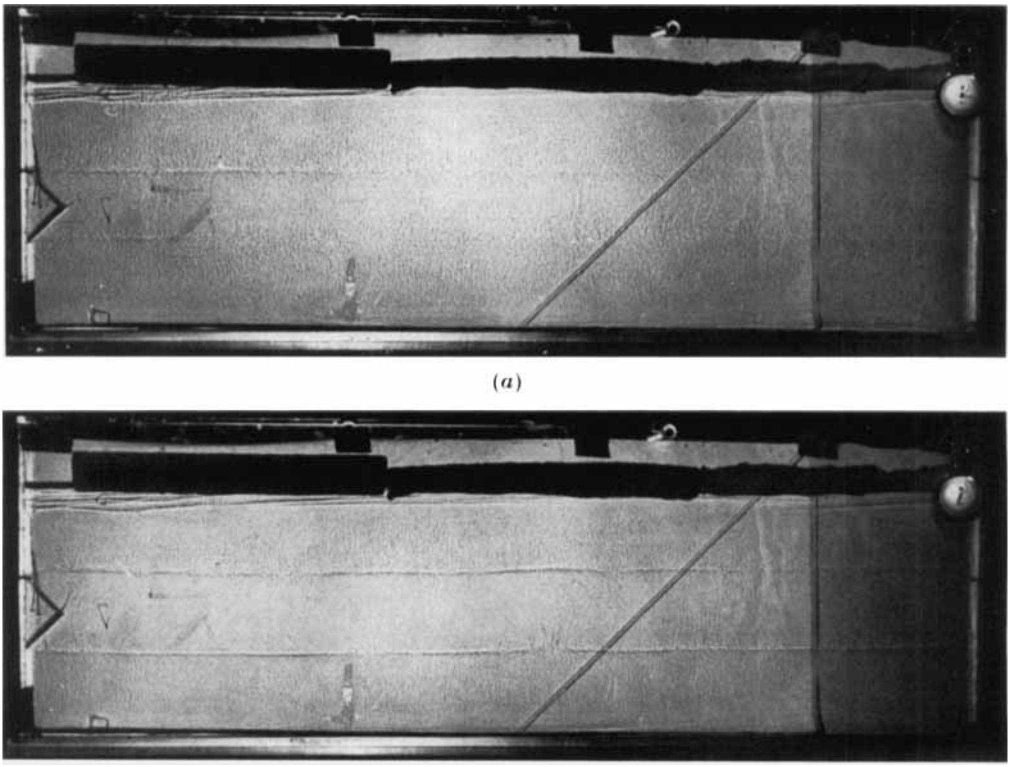


FIGURE 5. Finger case with $R_\rho = \beta\Delta S/\alpha\Delta T = 0{\cdot}97$, where ΔS and ΔT are the initial sugar and salt concentrations. A disturbance was introduced by closing the valve A connecting the supply tanks (figure 2) for 4 min during filling. Note the mixed layer advancing towards the left, below a surface layer whose formation is explained in the caption to figure 4. (The diagonal mark is a slot milled in the front and back walls of the tank, which is irrelevant except for the experiments described in § 5.)



(*a*)

(*b*)

FIGURE 6. Finger case, $R_\rho = 0{\cdot}97$. The disturbance was introduced by filling the tank past a wedge fixed on one end wall. (*a*) 39 min into the experiment, showing initiation of the convecting layer. (*b*) 63 min, with convection well established.

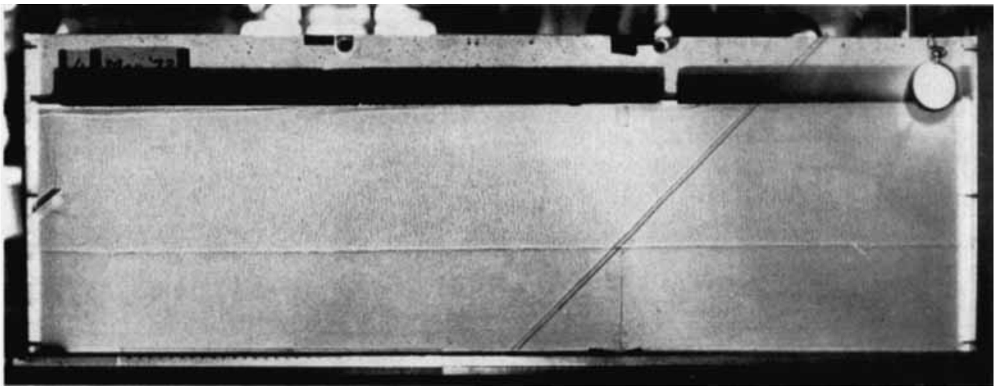


FIGURE 7. Finger case, $R_\rho = 0{\cdot}92$. A convecting layer was generated by setting the flap at 45° in a downward sloping position at 20 min. The flap was moved to the upward sloping position shown at 54 min, and the photograph taken at 85 min.

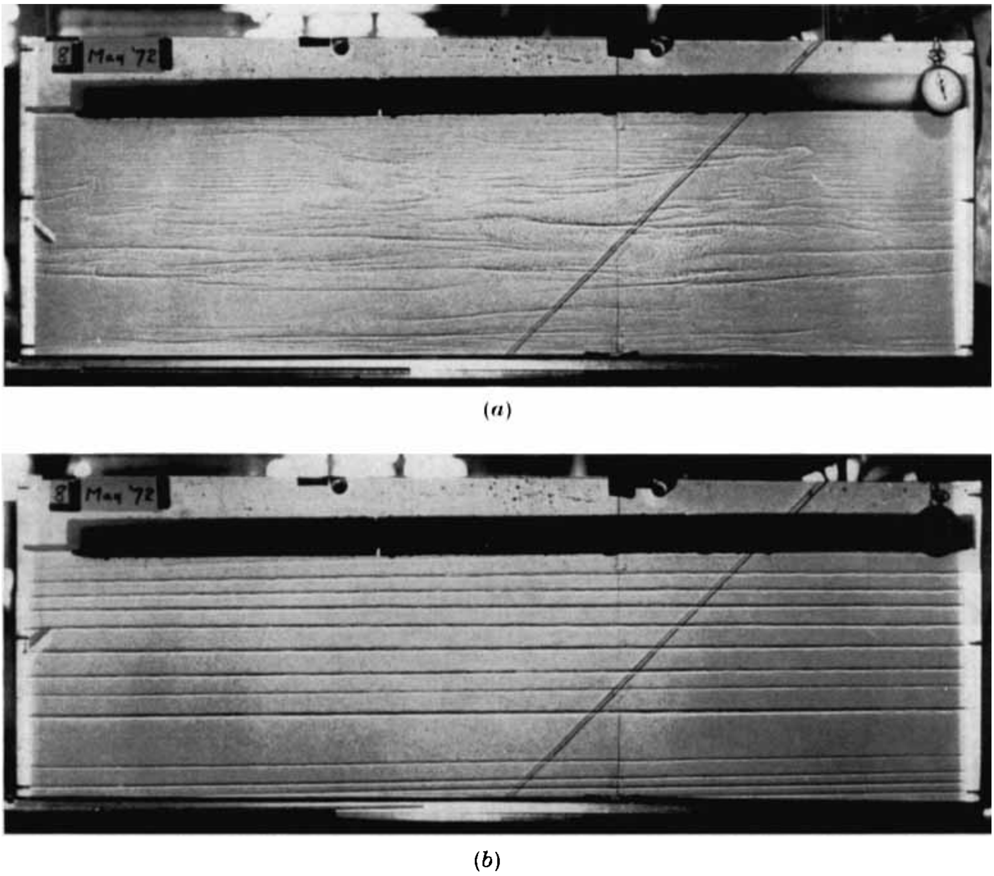
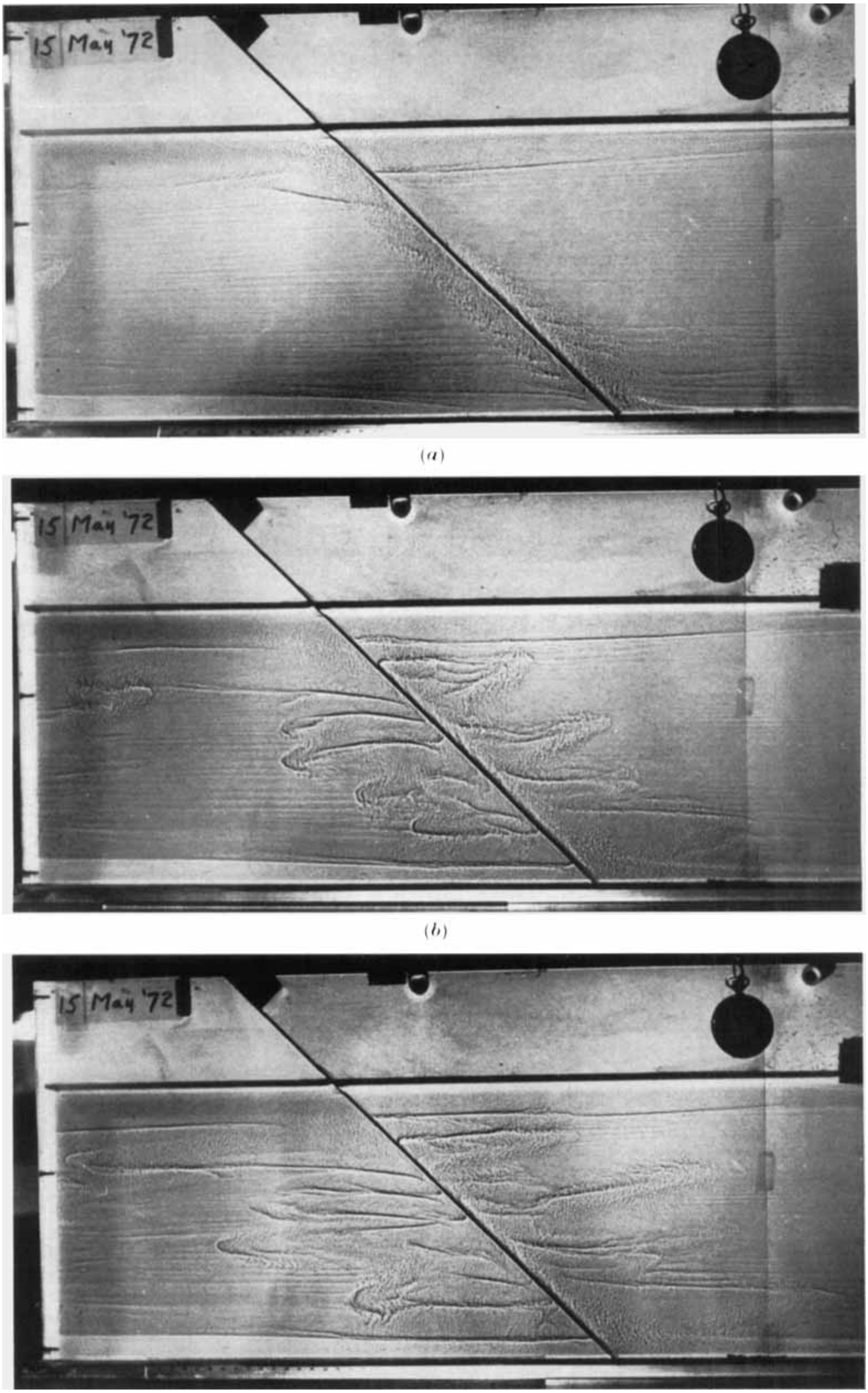


FIGURE 8. Diffusive case, $R_\rho = 1{\cdot}09$. Disturbances introduced inadvertently by filling through three floats. (*a*) 62 min. (*b*) 22 hr 10 min.

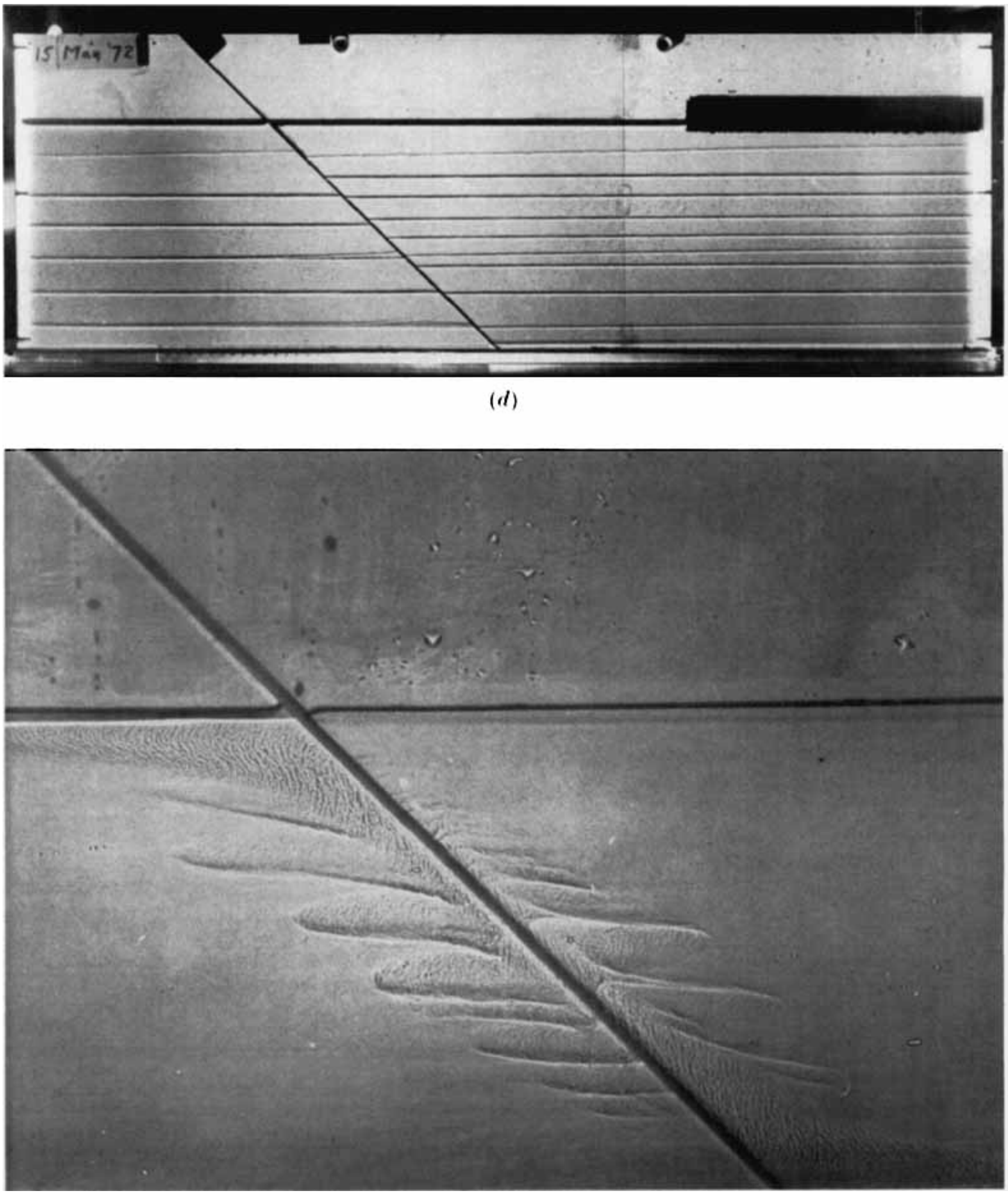


(*a*)

(*b*)

(*c*)

Figures 9 (*a–c*). For legend see plate 5.



(*d*)

(*e*)

FIGURE 9. Diffusive case, $R_\rho = 1{\cdot}09$; layer generation due to a sloping boundary. In the following, $t = 0$ is taken as the time when the boundary was introduced. (*a*) 10 min. Initiation. (*b*) 41 min. Outflow. (*c*) 49 min. Extension of layers. (*d*) 42 hr. Final stage. (*e*) An intermediate stage of a similar experiment, in which the layers were more regularly spaced.

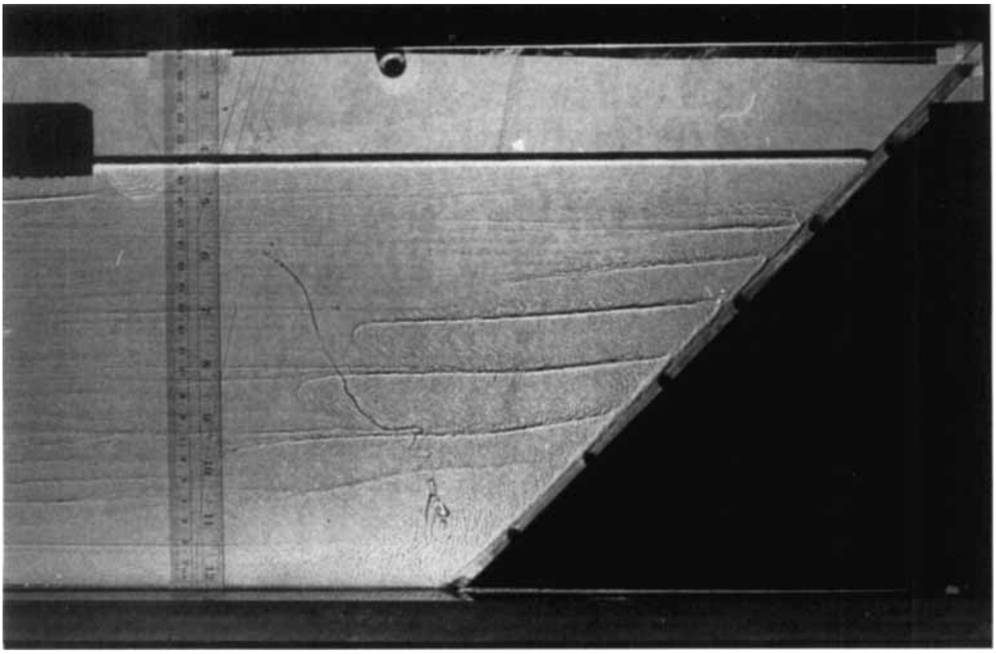
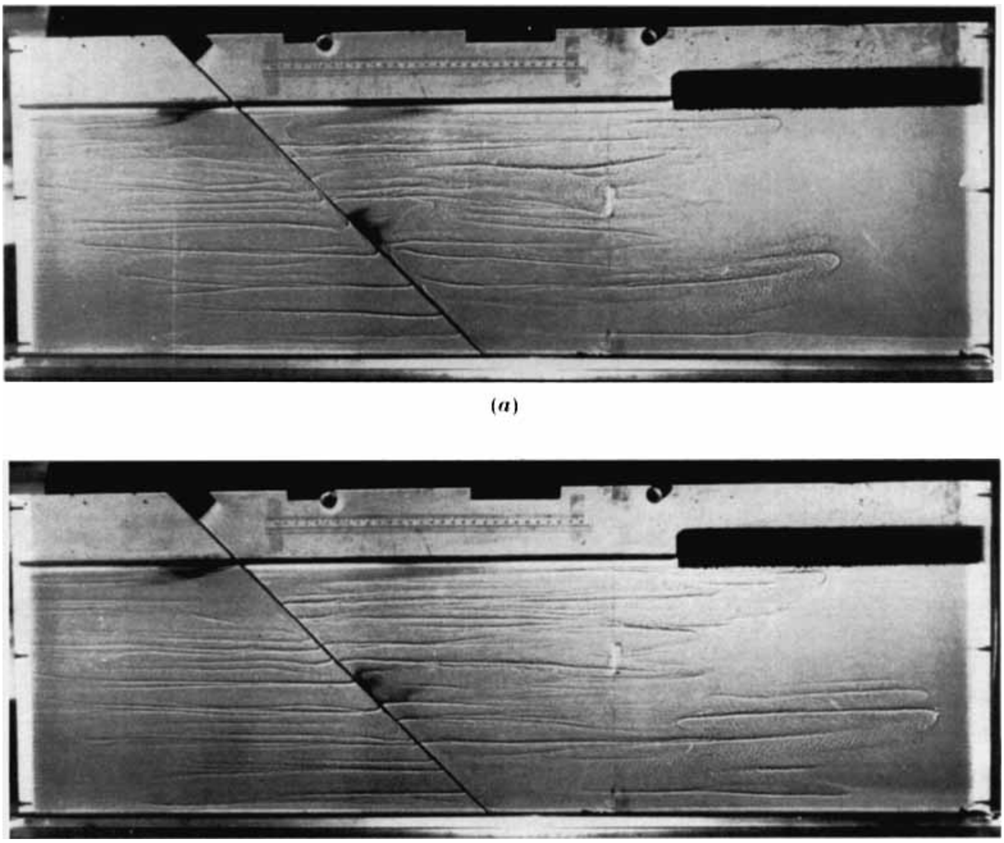


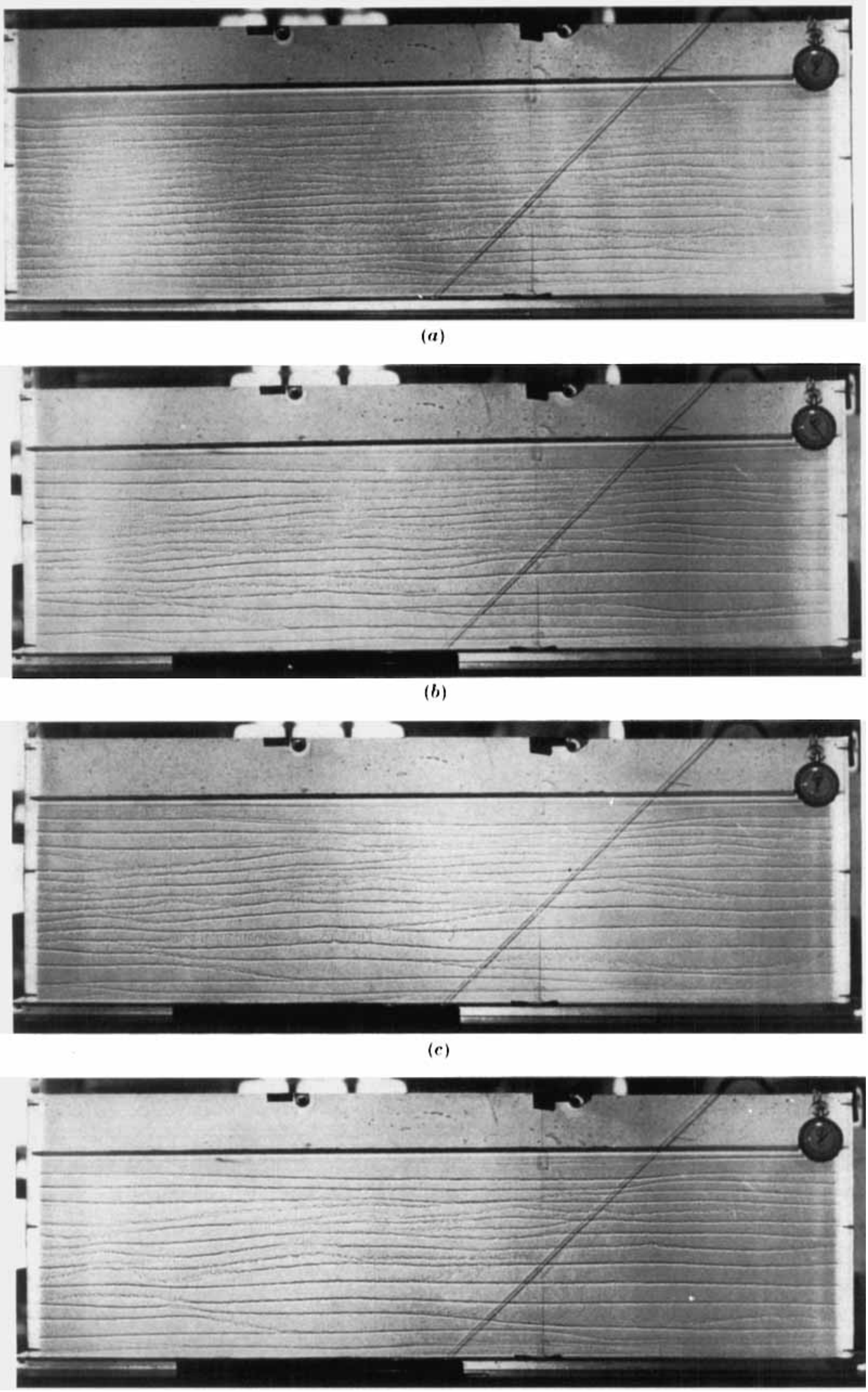
FIGURE 10. Layers formed by filling a tank in the diffusive sense past a stationary sloping boundary. Photograph 1 hr 30 min after starting to fill. (Please disregard the irregular sloping line, which is due to a glue mark on the tank wall.)



(*a*)

(*b*)

FIGURE 11. Diffusive case, $R_\rho = 1{\cdot}08$, showing fronts on right moving independently of the sloping boundary. (*a*) 1 hr 15 min. (*b*) 1 hr 30 min. (Times after the insertion of the plate.)



(*a*)

(*b*)

(*c*)

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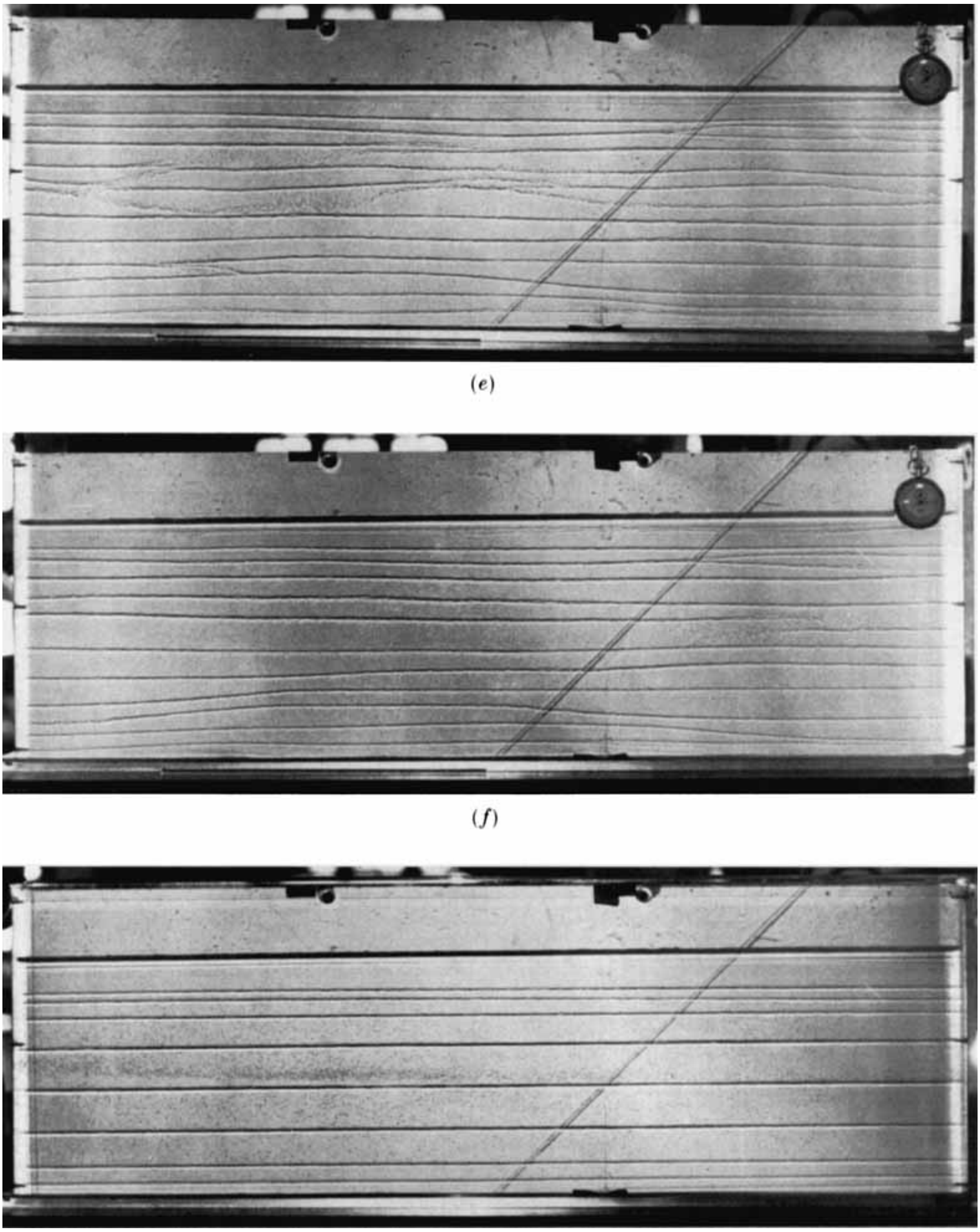
FIGURES 12 (*a–d*). For legend see plate 9.



(*e*)

(*f*)

(*g*)

FIGURE 12. Diffusive case, $R_\rho = 1{\cdot}04$, showing the process of merging of a series of thin layers. (*a*) 1 hr 38 min. (*b*) 2 hr 10 min. (*c*) 2 hr 25 min. (*d*) 2 hr 45 min. (*e*) 3 hr 13 min. (*f*) 3 hr 45 min. (*g*) 24 hr 45 min.

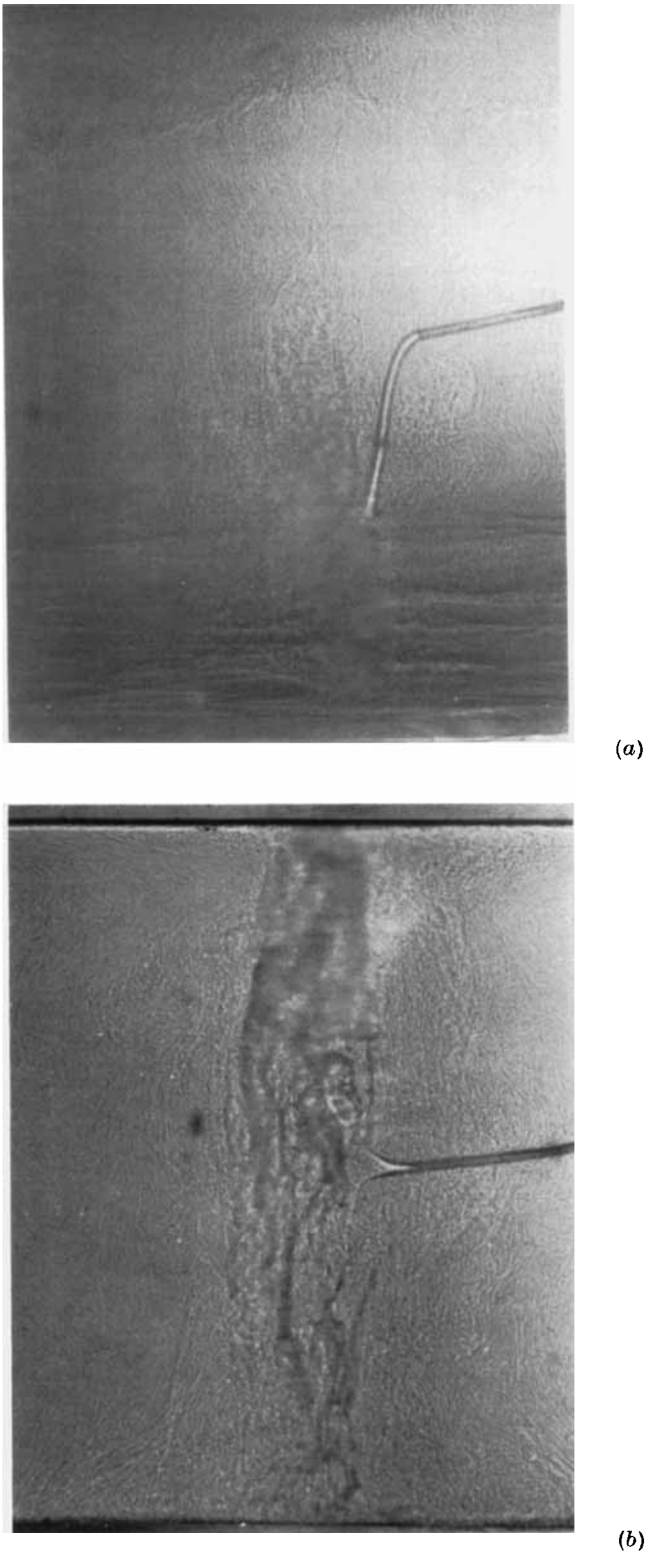


FIGURE 13. Showing the strong convective motions and layering produced by introducing a small source of sugar solution (specific gravity = 1·188) into an initially uniform tank of salt solution of almost the same density. (*a*) Sugar slightly heavy. (*b*) Adjusted to same density.

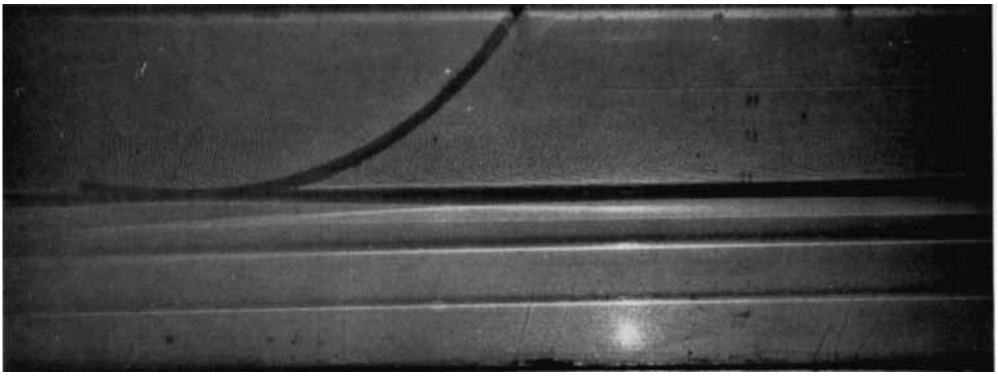


FIGURE 14. Fingers and layers produced in a long tank by introducing small sources of sugar (specific gravity = 1·094) and salt (specific gravity = 1·097) at opposite ends, and withdrawing the mixture at the centre. The photograph was taken at the left-hand end near the source of salt, when a steady state had been reached.